

Application of Aging and Dependence Concepts in Reliability Analysis

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Abstract

Many aging and dependence concepts are proposed in reliability literature. They have shown to be useful in different situations. However, most of the results are theoretical in nature, and how they applied to practical application and decision making is not clear. Few practical models that distinguish between different classes of distributions exist, and most of classes are for monotonic aging. The aging for the common bathtub-shaped distribution is, for example, not monotonic. In this talk, several practical issues ranges from product design and maintenance decision related to the use of aging and dependence concepts will be discussed. Hopefully further research can be geared towards models for practical reliability analysis.

1. Introduction

Various aging and dependence concepts have been proposed and discussed in reliability context (Barlow and Proschan, 1981, Belzunce et al., 1998, and Abouammoh and Qamber, 2003). However, most of the results are theoretical in nature, and how they can be applied to practical situations and in decision making is not clear. Few practical models that distinguish between different classes of distributions exist, and most of classes are for monotonic aging. The aging for the common bathtub-shaped distribution is, for example, not monotonic. Models describing this type of aging and the study of the effect of assuming no-aging should be investigated.

Another important concept in reliability is dependence as components in complex systems are not independent. How to model the dependence has attracted a lot of attention. We will share some different views in this context. A question that could be raised is how dependence concepts can be used in practice to improve the reliability. Some preliminary discussion will be provided in this paper.

In this paper, several practical issues ranges from product design and maintenance decision related to the use of aging and dependence concepts will be discussed. Hopefully further research can be geared towards models for practical reliability analysis.

2. Aging concepts and application

2.1 Basic notion of aging

There are numerous notions of aging and the basic reference is Barlow & Proschan (1981). The most commonly used ageing classes in reliability engineering are IFR and DFR and they are characterized by shapes of the failure rate function (hazard function or force of mortality).

For a lifetime distribution F , when the density function exists, the failure rate function is defined as

$$r(t) = \frac{f(t)}{1 - F(t)} \quad (1)$$

This function can be increasing or decreasing and it has the interpretation of the rate of failure pertaining to a lifetime distribution when the item has survived to time t .

A simple Weibull distribution can be used to model the monotonic increasing or decreasing failure rate. In industry, bathtub shaped failure rate function is commonly used, at least as illustration. The initial part of the failure rate is usually decreasing, and burn-in can be effective in screening out components that have early failures. The final part of the failure rate is usually increasing and it is due to fatigue or wear-out. Replacement is usually recommended. A research issue is to develop models for bathtub shaped failure rate function (Lai et al., 2003).

2.2. Relative aging

There are at least three types of relative aging of two systems (or components) A and B. The first of these is to compare how fast A ages in comparison with B. We say that A is aging faster B if the ratio of their failure rates $r_A(t)/r_B(t)$ is increasing in t . Lai and Xie (2004) consider the relative aging for two parallel systems. In this connection, Xie and Lai (1996) show that adding parallel redundancy on an IFR component is less effective than that for a DFR component.

There are numerous papers dealing with optimization and estimation issues. However, the physical aging properties are not easily explained, not to say how to reduce the aging. Exponential distribution is commonly used, but it has no aging, so a question is when we can use it and how we can use it as approximation.

2.3. Exponential distribution – how useful is it?

Aging and exponential distribution are two contradicting issues. Exponential distribution assumes no aging and the failure rate function is constant. However, the question is whether exponential distribution could be a good approximation and when it is the case. One common idea is that the exponential distribution can be used in the flat portion of the bathtub curve. Hence, in modelling useful life period of a product, simple exponential distribution could be used.

In application context, maintenance is applied to systems. For the widely used age replacement model, the time to the occurrence of first failure can be modelled with exponential distribution. This is because when the system enters the wear-out region, the system would be replaced. Hence, we can use it to justify the constant failure rate assumption (Xie et al., 2000). In fact, if with regular maintenance (Martorell et al., 1996), the system still shows a clearly increasing failure rate, instead of concluding that exponential

distribution can not be used, we should study the effectiveness of the maintenance in this situation, and probably modify that.

2.4. Exponential distribution as a conservative approximation

The most important question in reliability engineering is not how reliable a system or component is. It is whether it has met the reliability requirement. If so, then the system can be released. Several results in the book by Barlow and Proschan (1981) can be used in this context. Further research on when exponential distribution can be used as conservative approximation is of great practical importance.

One example of practical implication is when simple Weibull distribution , $R(t) = \exp(-(t / \alpha)^\beta)$, is used. When the inverse of the mean time to failure (MTTF) is used as the failure rate for exponential distribution, the actual mission reliability will be higher than the computed reliability when the failure rate is increasing or the shape parameter is greater than one and this information can be useful to consumers.

3. Component dependence in complex systems

3.1 Dependence – good or bad?

Dependence or correlation usually causes a lot of statistical problems and when models assume independence, the interpretation becomes an important issue. Current research focuses more on the definition of dependence and type of dependence such as association and quadrant dependent. It is of interest to see how the concepts could have more practical implications.

One such example is with the simple type of dependence; dependence could be positive or negative. Usually components in a system are positively correlated because they are affected by the same factors such as power supply, vibration, humidity etc. However, statistically we may also have negative dependence. That is, when T_1 decreases, T_2 increases or vice versa. It would be good to identify situations where this happens. This is especially when we have parallel redundancy when we only need one component to be working. For safety critical systems, redundancy is probably the only method that could help achieving high reliability of the system. It is well-known that dependence (understood as positive dependence) causes problems.

Kotz et al. (2003) investigate how the degree of correlation affects the increase in the mean lifetime for parallel redundancy when the two components are positively quadrant dependent. Further research in this direction is needed.

3.2. Pseudo component with reliability greater than one

To achieve higher reliability than independent case, a pseudo component with reliability greater than one could be used. When two components form a parallel redundant unit, the system reliability is $R_{system} = 1 - (1 - R_1)(1 - R_2)$ for the independent case. When we have dependence that could be modelled with a pseudo component (such as power supply to both components), the reliability becomes $R_{system} = [1 - (1 - R_{1c})(1 - R_{2c})]R_c$ which is lower when R_c is less than one. In our formulation and to make use of negative dependence, we actually have a component with reliability greater than one.

3.3. Possibility of making use of dependence

As mentioned, positive dependence leads to lower reliability for parallel system and reduces the effectiveness of redundancy. However, negative dependence could lead to higher reliability for such systems. At the system design stage, negative dependence could be build-in into the system. A negative dependence parameter, which has to be assumed unknown, can be incorporated into the modelling. This parameter can also be estimated with test data.

How this can be achieved in practice is an interesting question. When selecting component for redundant system, one could purposely chose components that operated better in different environment, so that if one is not working well, the other will. A monitoring device could also have this effect. Similar situations can be found in dealing with human reliability where operators can complement each other to improve the effectiveness of redundancy.

4. Discussions

This paper has highlighted some important concepts, aging and dependence, in reliability modelling and reliability engineering. These issues usually complicate the models and make the analysis difficult, so that the practitioners have tried to avoid using them. However, it is pointed out in this paper that these are the concepts that have potential useful implications, and they should be considered in a more positive direction. Some preliminary discussions are also presented to illustrate the use of these concepts.

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